WAVES OF FINITE AMPLITUDE IN A HOT PLASMA

Yu. A. Berezin

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 26–32, 1965

A study is made of a plane shock wave of arbitrary strength propagating in a hot rarefied plasma across the magnetic field. The question of the propagation of nonstationary waves of finite but small amplitude under these conditions is examined.

Fairly detailed studies have been made of waves of finite amplitude in a cold rarefied plasma. The profile of such waves is formed as the result of nonlinear and dispersion effects, the dispersion effects being caused by electron inertia and plasma anisotropy. If the gas-kinetic pressure of the plasma is taken into account, then dispersion effects appear which are associated with the fact that the Larmor radius of the ions is finite. Stationary waves of small but finite amplitude propagating across the magnetic field in a hot plasma (when the gas-kinetic pressure p is comparable with the magnetic pressure $H^2/8\pi$) have been treated in [1, 2]. In [1] an isolated rarefaction wave was found in a hot plasma, instead of the compression wave characteristic of a cold plasma, and a qualitative picture of the shock wave structure was given. In [2] a study was made of a small-amplitude shock wave with the finite size of the ion Larmor radius taken into account. The present paper investigates the structure of shock waves of arbitrary strength which propagate across the magnetic field in a fairly hot rarefied plasma, and also examines nonstationary waves of finite but small amplitude excited in a plasma by a "magnetic piston" acting over a limited time interval.

NOTATION

p—gas-kinetic pressure; H-magnetic field; u, v—macroscopic velocities along the x and y axes; ρ—density; $m_e(m_i)$—mass of electron (ion); σ—plasma conductivity; Ω_H—ion-cyclotron frequency; V_A—Alfvèn velocity; c-velocity of light; γ—adiabatic exponent; V—specific volume; $\omega_{0e}(\omega_{0i})$—electron (ion) plasma frequency; s_0—velocity of sound.

1. Basic equations.

The initial system of equations consists of the equations of motion of the electron and ion components of the plasma, the equations of continuity and Maxwell's equations. The plasma is assumed to be quasi-neutral. The gas-kinetic pressure is introduced into the equations of motion, and this has a tensor character due to the fact that the ion distribution does not have spherical symmetry. We shall take the motion to be one-dimensional, i.e., all quantities depend on the x coordinate and time t only. The magnetic field is directed along the z axis. We shall write the basic system of equations in the form of laws of conservation of mass, momentum (along the x and y axes), energy and magnetic flux

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)=0\,,\qquad (\rho=\rho_i+\rho_e)\,,$$

$$\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}\Big\{p+\frac{H^2}{8\pi}+\rho u^2-$$

$$-\frac{p_i}{2\Omega_H}\frac{\partial v}{\partial x}+\frac{cm_e p_i}{8\pi e\Omega_H}\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial H}{\partial x}\right)\Big\}=0,$$

$$\frac{\partial}{\partial t}(\rho v)+\frac{\partial}{\partial x}\left(\rho uv+\frac{p_i}{2\Omega_H}\frac{\partial u}{\partial x}\right)=0 \qquad (1.1)$$

$$\frac{\partial}{\partial t}\left\{\frac{p}{\gamma-1}+\frac{H^2}{8\pi}+\frac{1}{2}\rho(u^2+v^2)+\frac{1}{2}\frac{m_i m_e c^2}{(4\pi e)^2\rho_i}\left(\frac{\partial H}{\partial x}\right)^2\right\}+$$

$$+\frac{\partial}{\partial x}\left\{u\left[\frac{\gamma p}{\gamma-1}+\frac{1}{2}\rho(u^2+v^2)+\frac{1}{2}\frac{m_i m_e c^2}{(4\pi e)^2\rho_i}\left(\frac{\partial H}{\partial x}\right)^2+\right.\right.$$

$$\left.+\frac{H^2}{4\pi}-\frac{p_i}{2\Omega_H}\frac{\partial}{\partial x}\left(v-\frac{cm_e}{4\pi e\rho}\frac{\partial H}{\partial x}\right)\right]+\frac{p_i}{2\Omega_H}\left(v-\frac{cm_e}{4\pi e\rho}\frac{\partial H}{\partial x}\right)\frac{\partial u}{\partial x}-$$

$$\left.-\frac{m_i m_e c^2}{(4\pi e)^2}H\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)\left(\frac{1}{\rho}\frac{\partial H}{\partial x}\right)-\frac{c^2}{16\pi^2\sigma}H\frac{\partial H}{\partial x}\right\}=0\,,$$

$$\frac{\partial H}{\partial t}+\frac{\partial}{\partial x}(uH)-\frac{m_i m_e c^2}{4\pi e^2}\frac{\partial}{\partial x}\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)\left(\frac{1}{\rho}\frac{\partial H}{\partial x}\right)=\frac{c^2}{4\pi\sigma}\frac{\partial^2 H}{\partial x^2}\,,$$

$$v=\frac{1}{\rho}(\rho_i v_i+\rho_e v_e),\quad p=p_i+p_e,\quad \Omega_H=\frac{eH}{m_i c}\,. \qquad (1.1)\ \text{(cont'd.)}$$

Here ρ is the density of the plasma, u is the x component of the macroscopic velocity, v is the y component of velocity, p is the pressure, σ is the plasma conductivity which we will take as constant.

If the plasma conductivity is large, then the equations of state of the electron and ion gases will not be much different from the adiabatic equations with an effective adiabatic exponent $\gamma = 2$ (since we are considering motion across the magnetic field). Thus we may set $p_i = \alpha p$ with a sufficient degree of accuracy, where α = const is the ratio of the ion gas pressure to the total plasma pressure.

To find the wave dispersion law we linearize the system of equations (1.1) as usual and seek a solution in the form of plane waves, as a result of which we obtain

$$\left(\frac{\omega}{k}\right)^2=s_0^2+V_A^2\left\{\frac{1}{1+c^2k^2/\omega_{0e}^2}+\left(\frac{c}{\omega_{0i}}\right)^2\left(\frac{2\pi\alpha p_0}{H_0^2}\right)^2k^2\right\},$$

$$s_0=\sqrt{2p_0/\rho_0},\qquad V_A=\frac{H_0}{\sqrt{4\pi\rho_0}}\,,$$

$$\omega_{0e}=\sqrt{4\pi\rho_0 e^2/m_i m_e}\,,\qquad \omega_{0i}=\sqrt{4\pi\rho_0 e^2/m_i^2}\,, \qquad (1.2)$$

where p_0 and ρ_0 are the unperturbed pressure and density of the plasma, respectively, and s_0 is the velocity of sound.

If the gas-kinetic ion pressure is small enough compared with the magnetic pressure, the first term in the brackets in expression (1.2) predominates, resulting in negative dispersion (the phase velocity of small perturbations decreases as the wavelength decreases); in this case, as is well known, the characteristic linear dimension of the stationary compression waves is of the order of c/ω_{0e}. However, if the gas-kinetic pressure of the ions is high enough (hot plasma), then positive dispersion occurs (the phase velocity of small oscillations increases as the wavelength decreases); in this case the characteristic dimension of stationary rarefaction waves is of the order

$$\frac{c}{\omega_{0i}} \frac{2\pi\alpha p_0}{H_0^2} \gg \frac{c}{\omega_{0e}}$$

as is clear from expression (2.2).

Thus, in a hot plasma dispersion effects are basically caused by the finite size of the Larmor radius and not by the electron inertia.

We reduce the system of equations (1.1) to dimensionless form:

$$\frac{\partial}{\partial\tau}\frac{1}{V} + \frac{\partial}{\partial\xi}\frac{u}{V} = 0,$$

$$\frac{\partial}{\partial\tau}\frac{u}{V} + \frac{\partial}{\partial\xi}\left\{p + \frac{1}{2}h^2 + \frac{u^2}{V} - \frac{\alpha p}{2h}\frac{\partial}{\partial\xi}\left(v - \beta^2 V\frac{\partial h}{\partial\xi}\right)\right\} = 0,$$

$$\frac{\partial}{\partial\tau}\frac{v}{V} + \frac{\partial}{\partial\xi}\left(\frac{uv}{V} + \frac{\alpha p}{2h}\frac{\partial u}{\partial\xi}\right) = 0,$$

$$\frac{\partial}{\partial\tau}\left\{p + \frac{1}{2}h^2 + \frac{u^2+v^2}{2V} + \frac{1}{2}\beta^2 V\left(\frac{\partial h}{\partial\xi}\right)^2\right\} +$$

$$+ \frac{\partial}{\partial\xi}\left\{u\left[2p + \frac{u^2+v^2}{2V} + h^2 + \frac{1}{2}\beta^2 V\left(\frac{\partial h}{\partial\xi}\right)^2 -\right.\right.$$

$$\left. - \frac{\alpha p}{2h}\frac{\partial}{\partial\xi}\left(v - \beta^2 V\frac{\partial h}{\partial\xi}\right)\right] + \frac{\alpha p}{2h}\left(v - \beta^2 V\frac{\partial h}{\partial\xi}\right)\frac{\partial u}{\partial\xi} -$$

$$\left. - \frac{\omega_{0i}^2}{4\pi\Omega_{\varkappa}\sigma}h\frac{\partial h}{\partial\xi} - \beta^2 h\left(\frac{\partial}{\partial\tau} + u\frac{\partial}{\partial\xi}\right)\left(V\frac{\partial h}{\partial\xi}\right)\right\} = 0,$$

$$\frac{\partial h}{\partial\tau} + \frac{\partial}{\partial\xi}(uh) - \beta^2\frac{\partial}{\partial\xi}\left(\frac{\partial}{\partial\tau} + u\frac{\partial}{\partial\xi}\right)\left(V\frac{\partial h}{\partial\xi}\right) = \frac{\omega_{0i}^2}{4\pi\Omega_{\varkappa}\sigma}\frac{\partial^2 h}{\partial\xi^2},$$

$$\xi = \frac{x\omega_{0i}}{c}, \quad h = \frac{H}{H_0}, \quad V\ \beta^2 = \frac{\rho_0}{\rho},$$

$$\beta^2 = \frac{m_e}{m_i}, \quad \tau = \frac{V_A\omega_{0i}}{c}t. \tag{1.3}$$

Here the velocity and pressure are normalized by the Alfvèn velocity V_A and the quantity $\rho_0 V_A^2$, respectively, and these quantities are represented by the same symbols as before.

2. Stationary motions. We shall consider stationary plasma motions on the basis of the system of equations (1.3), and in order to do this we pass as usual to a coordinate system attached to a wave moving with constant velocity. In this coordinate system the plasma moves in the positive direction of the x axis. For stationary motions we have the equations

$$u = jV,$$

$$p + \frac{1}{2}h^2 + j^2 V - \frac{\alpha p}{2h}\frac{d}{d\xi}\left(v - \beta^2 V\frac{dh}{d\xi}\right) = C_1,$$

$$uv + \frac{\alpha p V}{2h}\frac{du}{d\xi} = 0,$$

$$u\left[2p + \frac{u^2+v^2}{2V} + h^2 + \frac{1}{2}\beta^2 V\left(\frac{dh}{d\xi}\right)^2 -\right.$$

$$- \frac{\alpha p}{2h}\frac{d}{d\xi}\left(v - \beta^2 V\frac{dh}{d\xi}\right) -$$

$$\left. - \beta^2 h\frac{d}{d\xi}\left(V\frac{dh}{d\xi}\right)\right] + \frac{\alpha p}{2h}\left(v - \beta^2 V\frac{dh}{d\xi}\right)\frac{du}{d\xi} - \tag{2.1}$$

$$- \frac{\omega_{0i}^2}{4\pi\Omega_{\varkappa}\sigma}h\frac{dh}{d\xi} = jC_2, \tag{2.1 cont'd.}$$

$$u\left[h - \beta^2\frac{d}{d\xi}\left(V\frac{dh}{d\xi}\right)\right] - \frac{\omega_{0i}^2}{4\pi\Omega_{\varkappa}\sigma}\frac{dh}{d\xi} = j.$$

$$j = \text{const}, \quad C_1 = p_0 + {}^1/_2 + j^2, \quad C_2 = 2p_0 + {}^1/_2 j + 1.$$

Here j is the mass flux.

For convenience, in the analysis which follows we shall transform the system of equations (2.1) to equations solved with respect to their first derivatives in order to obtain the direction field. We shall take the plasma to be fairly hot ($8\pi p / H^2 \gg m_e / m_i$), which will allow us to neglect the electron inertia. Then the system of equations (2.1) assumes the form

$$\frac{dV}{d\xi} = -\frac{2h}{\alpha p}v,$$

$$\varkappa\frac{dh}{d\xi} = Vh - 1 \quad \left(\varkappa = \frac{\omega_{0i}^2}{4\pi\Omega_{\varkappa}\sigma_j}\right), \tag{2.2}$$

$$\frac{dv}{d\xi} = \frac{2h}{\alpha p}\left(p + \frac{1}{2}h^2 + j^2 V - C_1\right).$$

$$p = V^{-1}\{C_2 - h + {}^1/_2\, v^2 - (C_1 - {}^1/_2\, j^2 V - {}^1/_2 h^2)\, V\}. \tag{2.3}$$

The equilibrium states 1 (ahead of the wave front) and 2 (behind the wave front) are determined by the singular points of Eqs. (2.2). Setting the right sides of these equations equal to zero, we obtain:

in the unperturbed state (ahead of the front),

$$v = v_1 = 0, \quad V = V_1 = 1, \quad h = h_1 = 1, \tag{2.4}$$

in the perturbed state (behind the front),

$$v = v_2 = 0, \quad h = h_2 = V_2^{-1},$$

$$V = V_2 = \frac{2(1 + 2p_0) + j^2}{3j^2}. \tag{2.5}$$

If $j^2 < 1 + 2p_0$, then $V_2 > 1$, and no wave joining the two different states exists (such a wave would be a rarefaction wave). In this case the steady-state solution is the isolated wave found in [1], which unites two identical states. We note that in [1] the basic equation describing the stationary wave is given for the case $\gamma \neq 2$, although $\gamma = 2$ for the motions of a collisionless plasma across the magnetic field, and so we can not pass directly from the results of paper [1] to the case which interests us. For small amplitudes it is not difficult to obtain

$$h = 1 - |\psi_{\max}|\,\mathrm{sch}^2\left\{\frac{s}{\alpha p_0}\sqrt{|\psi_{\max}|}(\xi - w\tau)\right\}, \tag{2.6}$$

$$\psi = h - 1 \ll 1, \quad w = s(1 - {}^1/_2|\psi_{\max}|), \quad s = \sqrt{1 + 2p_0},$$

where w is the velocity of the isolated wave. The isolated wave is a rarefaction wave and its velocity is less than the velocity of sound. If $j^2 > 1 + 2p_0$ (the wave velocity is greater than the velocity of sound), then $V_2 < 1$, and a shock wave occurs which joins two progressive plasma fluxes having different parameter values and propagates without change of profile at a certain constant velocity. In what follows we shall pay particular attention to the shock wave structure, i.e., to the case $j > 1 + 2p_0$.

We shall investigate the singular points of the system of equations (2.2). To do this we linearize Eqs. (2.2) close to their singular points, taking the departure of all quantities from their values in (2.4), (2.5) to be small, i.e., we assume

$$V = V_{1,2}(1+\varphi), \qquad h = h_{1,2}(1+\psi), \tag{2.7}$$

where $\psi, \varphi \ll 1$. Retaining quantities of the first order of smallness, we obtain

$$\frac{d\varphi}{d\xi} = -\frac{2h_{1,2}}{\alpha A_1 V_{1,2}} v, \qquad \varkappa h_{1,2}\frac{d\psi}{d\xi} = \varphi + \psi,$$
$$\frac{dv}{d\xi} = \frac{2h_{1,2}}{\alpha A_1}\left[(j^2 V_{1,2} - A_2)\varphi + h_{1,2}^2\psi\right], \tag{2.8}$$

$$A_1 = h_{1,2}\left[C_2 - h_{1,2} - (C_1 - j^2 V_{1,2} - {}^1/_2 h_{1,2}^2) V_{1,2}\right],$$
$$A_2 = h_{1,2}(C_2 - h_{1,2} - {}^1/_2 j^2 V_{1,2}^2). \tag{2.9}$$

Assuming a dependence $\varphi, \psi, v \sim \exp(\mu\xi)$, we obtain the characteristic equation

$$\varkappa h_{1,2}\mu^3 - \mu^2 + \left(\frac{2h_{1,2}}{\alpha A_1}\right)^2 \frac{j^2 V_{1,2} - A_2}{V_{1,2}} \varkappa h_{1,2}\mu -$$
$$-\left(\frac{2h_{1,2}}{\alpha A_1}\right)^2 h_{1,2}(j^2 V_{1,2} - A_2 - h_{1,2}^2) = 0. \tag{2.10}$$

The plasma conductivity is taken to be fairly large, so we can seek the roots of Eq. (2.10) in the form of a series in powers of $\varkappa$, i.e.,

$$\mu = \mu^{(0)} + \varkappa\mu^{(1)} + \ldots \tag{2.11}$$

Setting (2.11) in (2.10), we find the roots of the characteristic equation

$$\mu = \pm\frac{2h_{1,2}}{\alpha A_1}\left(\frac{h_{1,2}^2 + A_2 - j^2 V_{1,2}}{V_{1,2}}\right)^{1/2} + \frac{2\varkappa}{(\alpha A_1 V_{1,2}^3)^2}. \tag{2.12}$$

The third root is equal to

$$\mu = \frac{1}{\varkappa h_{1,2}} - \frac{4\varkappa}{(\alpha A_1 V_{1,2}^3)^2} \gg 1. \tag{2.13}$$

For the perturbed state the roots of the characteristic equation are real and different and two of them have different signs; thus the singular point corresponding to the perturbed state is a generalized saddle point (see [3]), and the integral curve goes into the singular point $V = V_2$, $h = h_2$, $v = 0$ at sufficiently large values of ξ. For the unperturbed state the roots of (2.12) are equal to

$$\mu = \pm i\frac{2}{\alpha p_0}\sqrt{j^2 - 1 - 2p_0} + \frac{2\varkappa}{(\alpha p_0)^2}. \tag{2.14}$$

The real parts of the roots differ from zero and have the same sign. Thus for negative ξ the singular point $V = h = 1$, $v = 0$, corresponding to the unperturbed plasma state ahead of the shock wave, is a generalized node point, and the integral curves approach the singular point under consideration asymptotically, "twisting" around it.

If we pass to the case of an ideal plasma ($\sigma = \infty$, $\varkappa = 0$), then the roots of the characteristic equation for the unperturbed state are purely imaginary:

$$\mu = \pm i\frac{2}{\alpha p_0}\sqrt{j^2 - 1 - 2p_0}.$$

It follows from the theory of differential equations [3] that a singular point may, in the presence of imaginary roots, be both a center (the integral curves are closed curves which circle around the singular point without passing through it), and also a focus (the integral curves "twist" around the singular point, approaching it asymptotically). To determine what sort of singular point we have in this case, we must take into account terms of the next order of smallness, which we rejected in obtaining the system (2.8). We may then write the necessary equations (for an ideal plasma) in the form

$$\frac{1}{2}\alpha p_0\frac{d\varphi}{d\xi} = -v, \quad \frac{1}{2}\alpha p_0\frac{dv}{d\xi} =$$
$$= (j^2 - 1 - 2p_0)\varphi + f(\varphi^2, \varphi^3, \ldots, v^2), \tag{2.15}$$

where the function f contains terms of the order of φ^2, $\varphi^3, \ldots$, and v^2. Equations (2.15) are symmetrical relative to the φ axis (or V), i.e., they are invariant with respect to the transformation $\xi \to -\xi$, $v \to -v$. Thus, in accordance with Poincaré's theorem [3], the singular point $V = 1$, $(\varphi = 0)$, $v = 0$, corresponding to the unperturbed state of an ideal plasma, is a center. Thus it follows that in the case of a plasma with infinite conductivity the integral curve leaving the singular point corresponding to the perturbed state never reaches the singular point corresponding to the unperturbed state. Thus in an ideal plasma there is an infinite train of undamped periodic waves; naturally such a structure cannot be called a shock wave, as it was in [2].

If $\sigma \neq \infty$, $(\varkappa \neq 0)$, then the amplitude of the periodic waves will be damped as they move towards the unperturbed plasma (for $\xi \to -\infty$). Such a structure is a shock wave joining two different states, while the shock wave region (more exactly, its leading edge) has an oscillatory structure, so that the shock wave in a hot plasma bears a qualitative resemblance to the shock wave in a collisionless cold plasma, propagating at an angle to the magnetic field, as was noted in [1].

We shall consider the structure of the shock wave close to the equilibrium states $V = h = 1$, $v = 0$ and $V = V_2$, $h = V_2^{-1}$, $v = 0$, linearizing and using the system of equations (2.8). Close to the unperturbed state the shock wave profile starts from small oscillations whose amplitude gradually increases. For this part of the profile (for $\xi < 0$) we may write

$$h(\xi) = 1 + Ce^{\delta\xi}\cos k\xi,$$

$$V(\xi) = 1 + C\frac{(\varkappa\delta - 1)\cos k\xi + \varkappa k\sin k\xi}{(\varkappa\delta - 1)^2 + \varkappa^2 k^2}e^{\delta\xi},$$

$$v(\xi) = -{}^1/_2\alpha p_0 C(\delta\cos k\xi - k\sin k\xi)e^{\delta\xi}, \tag{2.16}$$

$$C = \text{const}, \quad \delta = \frac{2\varkappa}{(\alpha p_0)^2}, \quad \lambda \sim \frac{2\pi}{k} = \frac{\pi\alpha p_0}{\sqrt{j^2 - 1 - 2p_0}}.$$

Here δ is the growth increment of the amplitude oscillations, λ is the wavelength of the oscillations.

Thus it is clear that when the unperturbed pressure is increased, significant damping appears over a greater length, and the linear dimension of the oscillations increases. We note that the magnitude of the damping is determined by the conductivity and pressure alone, and does not depend on the velocity of the shock wave, while the dimension of the oscillations depends significantly on the shock wave velocity, being inversely proportional to this velocity for $j^2 \gg 1 + 2p_0$. Close to the equilibrium perturbed state (for $\xi > 0$) the shock wave profile is described by the formulas

$$V(\xi) = V_2\{1 + C(1 - \varkappa\mu h_2)e^{\mu\xi}\},$$

$$h(\xi) = h_2(1 - Ce^{\mu\xi}),$$

$$v(\xi) = -\frac{\alpha A_1}{2h_2} C\mu V_2(1 - \varkappa\mu h_2)e^{\mu\xi}. \quad (2.17)$$

Here C > 0 is an arbitrary constant, A_1 is determined by expression (2.9), and μ by expression (2.12) with a minus sign in front of the first term.

The complete structure of a shock wave of arbitrary strength may be found by solving the system of equations (2.2). This system was solved numerically, values of the specific volume V, the magnetic field h and the transverse velocity v, calculated from formulas (2.17) for a certain fairly large positive $\xi = \xi_0$ being chosen as the initial conditions. Thus system (2.2) was solved from a point ξ_0, close to the equilibrium perturbed state, to a point $\xi = \xi_{max}$, where the amplitudes of the required functions approached sufficiently close to the values corresponding to the unperturbed equilibrium state. By way of example, the profile of a shock wave is given in the figure for the following values of the parameters $p_0 = 0.4$, $j = 2$, $\alpha = 0.5$. In this case the total length of the shock transition region is roughly 10 c/ω_{0i}. As the unperturbed ion pressure increases, this region is extended, and the linear dimension of the oscillations increases.

3. Nonstationary waves. We shall now consider nonstationary waves of finite but small amplitude, propagating across the magnetic field in a hot ideal plasma. For weak waves, assuming $V = 1 + \varphi$, $h = 1 + \psi$ (φ, $\psi \ll 1$) and retaining in Eqs. (1.3) terms up to and including those of the second order of smallness, in deviations from the unperturbed values, just as in [4] for waves in a cold plasma, we obtain the equation

$$\frac{\partial\psi}{\partial\tau} + s\frac{\partial\psi}{\partial\xi} + \frac{3}{2}s\psi\frac{\partial\psi}{\partial\xi} + \frac{1}{2s}\left[\beta^2 - \left(\frac{\alpha p_0}{2}\right)^2\right]\frac{\partial^3\psi}{\partial\xi^3} = 0. \quad (3.1)$$

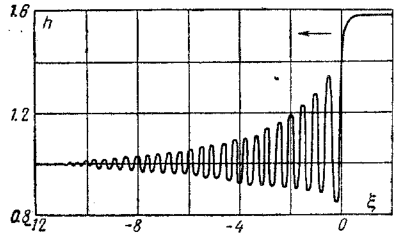
If we pass to the case of a cold plasma ($p_0 \to 0$), then $s \to 1$ and Eq. (3.1) coincides with Eq. (2.34) of [4] with an accuracy to the symbols involved. After the change of variables

$$\eta = \left(\frac{2s}{\nu}\right)^{1/3}(\xi - s\tau), \quad \nu = \beta^2 - \left(\frac{\alpha p_0}{2}\right)^2, \quad f = 3\left(\frac{s^4}{4\nu}\right)^{1/3}\psi, \quad (3.2)$$

Eq. (3.1) reduces to the form

$$\frac{\partial f}{\partial\tau} + f\frac{\partial f}{\partial\eta} + \frac{\partial^3 f}{\partial\eta^3} = 0. \quad (3.3)$$

A solution of this equation was found in [4], giving the asymptotic behavior of waves of finite but small amplitude excited by a "magnetic piston" acting on the plasma-vacuum boundary in the course of a limited time interval. The same solution also holds for waves



in a hot plasma. If we neglect electron inertia, then formulas obtained in [4] for waves propagating in a cold plasma at an angle θ to the magnetic field also apply to the case under consideration, while the quantity $(c/\omega_{0i})(2\pi\alpha p_0/H_0^2)$ plays the part of characteristic linear dimension instead of $c\theta/\omega_{0i}$.

In conclusion the author thanks R. Z. Sagdeev and N. N. Yanenko for discussing the paper, and also R. N. Makarov for helping with the numerical computations.

REFERENCES

1. V. E. Zakharov, "Stationary nonlinear waves in a finite-temperature plasma," PMTF, no. 6, 1964.

2. A. D. Pataraya, "The structure of weak shock waves taking into account the ion Larmor radius," Zh. tekhn. fiz., vol. 35, no. 2, 1965.

3. V. V. Nemytskii and V. V. Stepanov, Qualitative Theory of Differential Equations [in Russian], Gostekhizdat, 1949.

4. Yu. A. Berezin and V. I. Karpman, "Towards a theory of nonstationary waves of finite amplitude in a rarefied plasma," ZhETF, vol. 46, no. 5, 1964.

15 June 1965 Novosibirsk